A new way to look @ graphs. :)

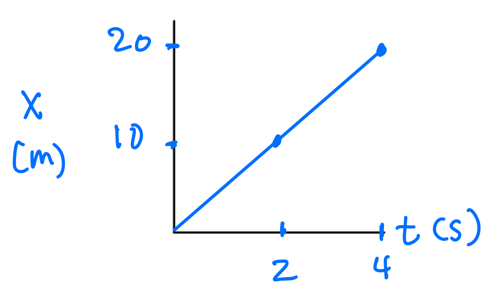
Let's imagine a car drives by at a constant speed of 5 m/s. It would be easy to calculate how far the car went in 2 seconds, or in 4 seconds, or any amount of time. We would simply say

$$d = \bar{v}\, t$$

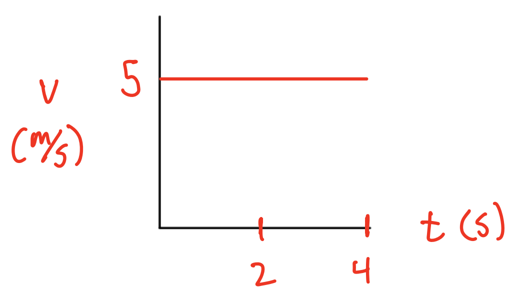
So in 2 seconds: $d = (5)(2) = 10\text{ m}$

in 4 seconds: $d = (5)(4) = 20\text{ m}$

We could also make graphs of Position vs. time and of velocity vs. time as shown below:



Position v. time



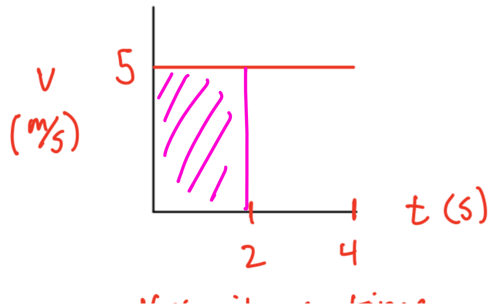
Velocity v. time.

You already know that the slope of the position graph is the velocity. So the red velocity graph is the slope of the blue position graph.

MATH SPOILER ALERT!

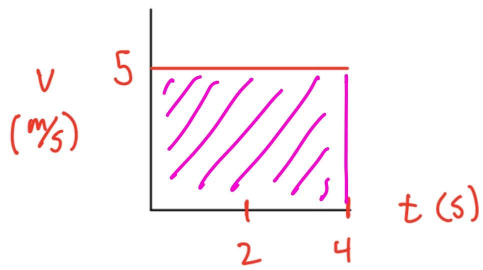
The fundamental idea of calculus is that if quantity A is the slope of the graph of quantity B, then quantity B is the area under the graph of quantity A.

For Example:



Notice that the pink rectangle has a "height" of 5 m/s and a "base" of 2 s. So its area is (5 m/s)(2 s) = 10 m.

Hey! That was the first calculation at the start of this document!



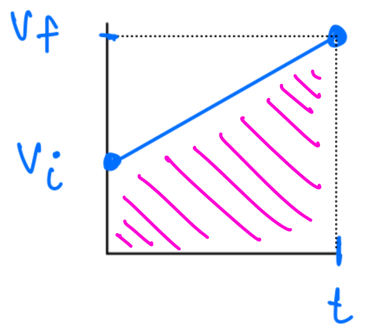
Velocity v. time.

Now this one has a height of 5 m/s and a base of 4 s. So its area is (5 m/s)(4 s) = 20 m. (which was the second calculation)

This means that the area under a velocity graph tells you the change in your position. This is true even if the velocity is not constant. Differential calculus will teach you how to find the slopes of almost any function and integral calculus will teach you how to find the area under most functions. It's really pretty cool. :)

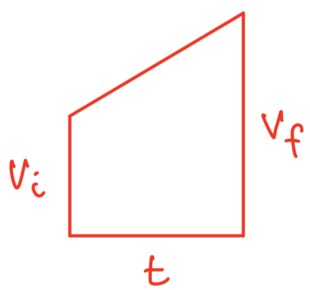
This idea is the key to seeing why we can say

$\bar{v} = \frac{v_i + v_f}{2}$ when the acceleration is constant.



This graph shows a constant acceleration from an initial velocity of v_i to a final velocity v_f. The area under that graph is shaded pink and represents the change in position (Δx) over that interval t.

The velocity graph makes a trapezoid with a base of "t" and the two heights of "v_i" and "v_f", shown below.



Therefore, the area of that trapezoid is

$$\text{Area} = \left(\frac{v_i + v_f}{2}\right) t$$

Since the area under the velocity graph is the change in position, we can say

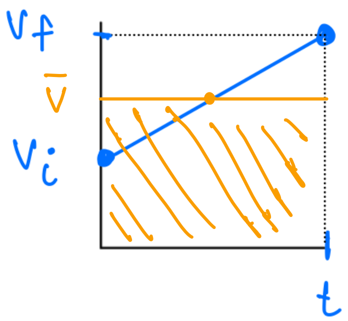
$$\text{Area} = \Delta x = \left(\frac{v_i + v_f}{2}\right) t$$

so that

$$\frac{v_i + v_f}{2} = \frac{\Delta x}{t}$$

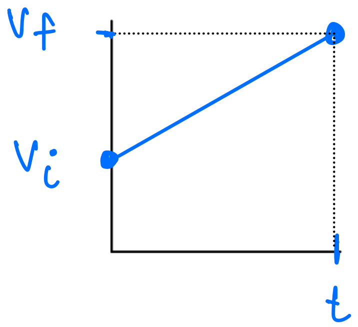
Hey! That's $\bar{v}$!

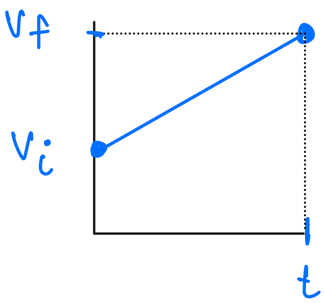
$$\therefore \quad \boxed{\bar{v} = \frac{v_i + v_f}{2}}$$

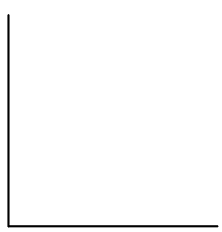


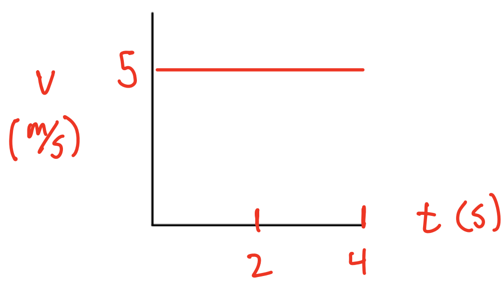
The average of v_i and v_f will be exactly in between v_i & v_f. Because the acceleration is constant, $\bar{v}$ will "cut" the line connecting v_i & v_f in half, making a two little identical triangles.

In the two diagrams above, the area of the pink trapezoid is the same as the area of the orange rectangle

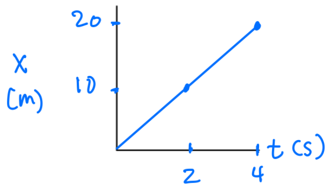




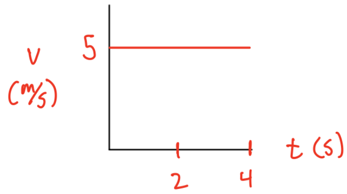




Velocity v. time.



Position v. time



Velocity v. time.

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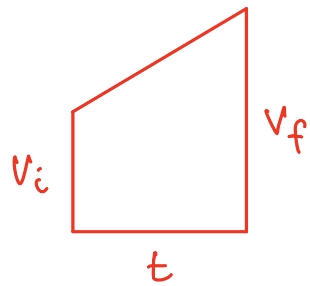

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Hey! That's $\bar{v}$!